

Then Ω , the solid angle shielded is given by

$$\begin{aligned}\Omega &= 2 \int_{\gamma-\beta}^{\alpha} \int_0^{F(\theta)} \sin \theta \, d\theta \, d\varphi \\ &= 2 \int_{\gamma-\beta}^{\alpha} \sin \theta \, F(\theta) \, d\theta.\end{aligned}$$

Finally we get the 'shielded fraction' by dividing by the total solid angle subtended by oxygen O(1).

$$\text{Shielded fraction} = \frac{\Omega}{2\pi(1-\cos\alpha)}.$$

Case 3. Large fraction shielded

If $\gamma < \beta < \alpha + \gamma$, overlap occurs and the width of the area shielded lies between the radius and the diameter of the atom being shielded. In this case the solid angle shielded = $A + B$,

$$\begin{aligned}\text{where } A &= \int_0^{\beta-\gamma} \int_0^{2\pi} \sin \theta \, d\theta \, d\varphi \\ &= 2\pi[1 - \cos(\beta - \gamma)]\end{aligned}$$

$$\text{and } B = 2 \int_{\beta-\gamma}^{\alpha} \sin \theta \, F(\theta) \, d\theta.$$

Then for the 'shielded fraction' we have

$$\text{'Shielded fraction'} = \frac{A+B}{2\pi(1-\cos\alpha)}.$$

Case 4. Complete shielding

If $\beta \geq \alpha + \gamma$ the oxygen atom is completely shielded and

$$\text{'Shielded fraction'} = 1.$$

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Neutron Diffraction Refinement of the Structure of Potassium Oxalate Monohydrate

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The structure of potassium oxalate monohydrate has been refined to a high degree of precision using three-dimensional neutron diffraction data. The conventional R value is 0.026 and the estimated standard deviations of atomic coordinates are between 0.001 and 0.003 Å. The oxalate ion is not quite planar with a separation of 0.037 (6) Å between the two parallel carboxyl planes. The C-C bond length is 1.581 (2) Å and the two C-O bond lengths are 1.262 (2) and 1.256 (2) Å. The O-H distance in the hydrogen bonds from the water molecule to the oxalate ion is 0.963 (3) Å; the O...O distance is 2.754 (2) Å, the O-H...O angle is 169.66 (22)° and the H-O-H angle, 107.62 (33)°. The root-mean-square amplitude of thermal motion of the water oxygen atom is considerably larger than that of the hydrogen atoms in some directions. The structure is compared with a recent X-ray structure of the same crystal and the agreement is excellent.

Introduction

In an earlier paper (Chidambaram, Sequeira & Sikka, 1964) (CSS) we reported a two-dimensional neutron

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diffraction study of the structure of potassium oxalate monohydrate. In that study the water molecule was shown to have a rather unusual type of coordination with neither of its two lone-pairs of electrons being

specifically directed. The water molecule in this crystal has recently also been the subject of extensive studies by proton magnetic resonance (McGrath & Paine, 1964; Pederson, 1966, 1968). McGrath & Paine observed that, unlike the case for $\text{Ba}(\text{ClO}_3)_2 \cdot \text{H}_2\text{O}$, $\text{Li}_2\text{SO}_4 \cdot \text{H}_2\text{O}$ or gypsum, the proton-doublet splitting in this crystal showed surprisingly little or no temperature dependence. In view of the unusual coordination and behaviour of the water molecule it seemed worth while to investigate this structure further using three-dimensional neutron diffraction data. Towards the completion of this investigation, however, an accurate X-ray diffraction study using diffractometer data (Hodgson & Ibers, 1969) (HI) came to our attention. The results of the X-ray study have been compared with the present results, the two being of comparable precision.

Experimental

Neutron intensity data were collected using the 4-circle neutron diffractometer 3-D FAD (Momin, Sequeira & Chidambaram, 1968) at the CIRUS reactor in Trombay. A cylindrical crystal specimen, 4.2 mm in diameter and 4.7 mm long (weight 126 mg) with its axis parallel to the crystallographic b axis, was prepared from a large single crystal grown by slow evaporation from a saturated aqueous solution. The crystal was dipped several times in liquid nitrogen so as to reduce extinction effects and mounted in the 'symmetric' position with its b axis parallel to the φ axis. The orientation of the specimen and the cell parameters were first refined on the basis of the optimized 2θ , χ and φ values of some 20 arbitrary reflexions, using our program *REFINE* (Srikanta, 1968*a*). The refined values of the cell constants (space group $C2/c$) agree well with the more reliable values obtained by HI which are as follows: $a=9.222$ (3), $b=6.197$ (2), $c=10.690$ (5) Å and $\beta=110.70$ (3)°. The intensities of 414 independent reflexions within the limit $\sin \theta/\lambda=0.575$ ($\lambda=1.406$ Å) were then recorded using the θ - 2θ coupled step-scanning technique. The scan-lengths ranged from 4 to 6° in steps of 0.1°. As the automation was not completed at the time, the recording was done manually at the rate of two reflexions per hour. Two standard reflexions were measured after every 20 reflexions to keep a check on the stability of the crystal and that of the counting equipment. The peak intensities of the $0k0$ reflexions showed little variation as a function of rotation about the scattering vector, indicating that the effects of multiple reflexions were small. It was not practicable to check the other reflexions for the effects of multiple reflexion.

The data were reduced using our program *DATARED* (Srikanta, 1968*b*). The signal counts were corrected for background in the standard way to obtain the net counts C_N , whose standard deviations [$\sigma(C_N)$] were estimated using the following relation (Busing & Levy, 1957)

$$\sigma^2(C_N) = [C_S + (N_S/N_B)^2 C_B + (0.1 C_N)^2],$$

where C_S is the total signal count in N_S steps and C_B is the background count in N_B steps. C_N and $\sigma(C_N)$ were then converted to F^2 and $\sigma(F^2)$ respectively, by applying the Lorentz and absorption corrections. An absorption coefficient of 0.75 cm^{-1} (measured) was used. The values of F^2 were averaged* for the few reflexions measured more than once.

Refinement

The atomic positions and isotropic temperature factors reported in the two-dimensional study of CSS were taken as the starting parameters for the refinement. An initial value of the scale factor was also estimated by comparison with the two-dimensional data. The least-squares refinement was carried out on F^2 with weights $\omega=1/\sigma^2(F^2)$, using the program *ORFLS* (Busing, Martin & Levy, 1962) incorporating modifications by Hamilton, Ibers & Johnson. Three cycles of isotropic refinement followed by three cycles of anisotropic refinement resulted in values of 0.061 and 0.117 respectively, for $R_1(=\Sigma||F_o|-|F_c||/\Sigma|F_o|)$ and $R_2(=\Sigma|F_o^2-F_c^2|/\Sigma F_o^2)$. At this stage it was obvious that the F_o^2 values of the intense reflexions were affected by extinction and hence an empirical extinction correction of the form

$$F_o^2(\text{corrected}) = F_o^2 \exp(gQ),$$

where $Q \equiv F_o^2(\text{corrected})/\sin 2\theta$, was applied. A value of 0.00202 for the constant g was obtained by plotting $\log(F_o^2/F_c^2)$ as a function of $F_c^2/\sin 2\theta$. This reduced R_1 and R_2 to 0.044 and 0.068 respectively in two more cycles of refinement. An error-analysis carried out at this stage gave the following relation for the average discrepancies between observed and calculated F^2 's:

$$\langle |\Delta(F^2)| \rangle_{\text{av}} = 0.15 + 0.05Q.$$

Two more cycles of anisotropic refinement using a weighting scheme $\omega^{-1/2} = \langle |\Delta(F^2)| \rangle_{\text{av}}$ and a revised value of $g=0.00218$ reduced R_1 and R_2 to 0.038 and 0.054. An examination of the data at this stage showed that for a few (11) weak reflexions the discrepancies ($|\Delta(F^2)|$) were greater than 5σ . These reflexions were rejected leaving 403 independent reflexions. The error-analysis was repeated ($\langle |\Delta(F^2)| \rangle = 0.138 + 0.0374Q$) and further anisotropic refinement was carried out omitting observations for which $F_o^2 < 2\sigma$. Finally the scattering amplitudes of K, O and C atoms were also allowed to vary, the scattering amplitude of H being fixed at $-0.378 \times 10^{-12} \text{ cm}$. Their values (in units of 10^{-12} cm) converged to 0.373 (8), 0.593 (10) and 0.675 (12) for K, O and C respectively. The final R values were as follows:

Number of observations	R_1	R_2	R_w^*
403	0.029	0.045	0.061
375 ($F_o > 2\sigma$)	0.026	0.045	0.060

* The mutual agreement in the intensities of the repeated reflexions was generally better than 1 per cent.

Table 1. Positional and thermal parameters for $K_2C_2O_4 \cdot H_2O$

The X-ray parameters are from Hodgson & Ibers, and our numbering system corresponds to theirs. The values of all the parameters have been multiplied by 10^5 , and the standard deviations are given in parentheses. The differences Δ are expressed in terms of the pooled error $\sigma_p \equiv (\sigma^2_{x-ray} + \sigma^2_{neutron})^{1/2}$.

K	x	y	z	β_{11}^*	β_{22}	β_{33}	β_{12}	β_{13}	β_{23}
Neutron	13202 (19)	83024 (27)	13207 (16)	681 (28)	1363 (64)	408 (20)	-12 (25)	194 (15)	103 (21)
	X-ray	13179 (3)	83038 (5)	13194 (3)	1331 (8)	427 (3)	-10 (4)	181 (3)	106 (3)
O(1)	Δ/σ_p	1-2	0-5	0-7	0-5	1-0	0-1	0-9	0-1
	Neutron	12622 (12)	26965 (19)	9153 (11)	770 (17)	589 (13)	-192 (16)	475 (11)	-207 (14)
O(2)	X-ray	12580 (10)	26910 (20)	9130 (10)	810 (10)	620 (10)	-190 (20)	470 (10)	-170 (10)
	Δ/σ_p	2-6	2-0	1-4	2-0	1-8	0-1	0-3	2-2
O(3)	Neutron	31972 (13)	48648 (17)	9262 (11)	866 (18)	534 (13)	-268 (17)	357 (11)	-235 (14)
	X-ray	31960 (10)	48690 (20)	9240 (10)	900 (10)	580 (10)	-260 (10)	320 (10)	-230 (10)
C	Δ/σ_p	0-8	1-6	1-4	0-8	2-9	0-4	2-5	0-3
	Neutron	0	49578 (35)	0-25	1755 (37)	916 (25)	0	918 (27)	0
H	X-ray	0	49740 (30)	0-25	1960 (40)	940 (20)	0	980 (30)	0
	Δ/σ_p	2-6	0-8	3-2	3-7	0-8	0	1-5	0
C	Neutron	23433 (10)	32553 (13)	5279 (8)	527 (15)	306 (11)	-55 (13)	213 (8)	-45 (11)
	X-ray	23470 (10)	32570 (20)	5320 (10)	590 (10)	336 (9)	30 (10)	192 (9)	10 (10)
H	Δ/σ_p	2-6	0-8	3-2	3-5	2-1	5-1	1-7	3-4
	Neutron	3497 (27)	40404 (37)	19367 (23)	1333 (39)	747 (31)	67 (36)	495 (22)	-105 (32)
H	X-ray	2200 (400)	42000 (400)	19900 (300)	2400 (600)	1200 (400)	-1000 (600)	90 (40)	40 (50)
	Δ/σ_p	3-2	4-0	1-8	1-8	1-1	1-8	9-5	2-5

* The form of the anisotropic temperature factor is $\exp [-(\beta_{11}h^2 + \beta_{22}k^2 + \beta_{33}l^2 + 2\beta_{12}hk + 2\beta_{13}hl + 2\beta_{23}kl)]$.

Discussion

The final positional and thermal parameters with standard deviations in parentheses are given in Table 1, where they are also compared with the corresponding values of the parameters found by HI. For the non-hydrogen atoms, the standard deviations of the parameters in the two sets are comparable and the overall agreement between them is excellent. The bond distances and bond angles are compared in Table 2; the O-H distance found by the X-ray method is too low, in agreement with the general trend in X-ray determinations. The final values of F_o^2 and F_c^2 are listed in Table 3.

The structure is as described by CSS and by HI. Numbering of the atoms in this paper corresponds to that of HI. A complete description of the coordination of the water molecule and its hydrogen bonding has been given by CSS and will not be repeated here.

Oxalate ion

The oxalate ion with its two parallel O(1)-C-O(2) planes separated by 0.037 (6) Å is not quite planar, although the sum of the bond angles around the carbon atom is 359.98 (16)°. The values of 1.5698 (17), 1.2559 (14) and 1.2487 (15) Å for the C-C, C-O(1) and C-O(2) bond lengths are in good agreement with the values of 1.5740 (24), 1.2595 (16) and 1.2473 (17) Å obtained by HI. The bond between the carbon atom and the hydrogen-bonded oxygen atom [C-O(1)] is somewhat longer than the other carbon-oxygen [C-O(2)] bond; a still larger difference between C-O(1) and C-O(2) was found by CSS in their earlier, less accurate two-dimensional neutron study and ascribed tentatively to the partial covalent character of the hydrogen bond for which O(1) is the acceptor. HI have also found a difference between the two C-O bonds and in the same direction, though of about twice the magnitude of that found in this investigation.

Motion of the water molecule

The mean-square amplitudes (u_{ij}) of thermal motion of the water oxygen and hydrogen atoms referred to the principal axes of the water molecule (X_1 along the H-H bond, X_2 along the twofold axes and X_3 along the plane normal) are given in Table 4.

The excellent agreement between the neutron and X-ray values of the thermal parameters of the non-hydrogen atoms suggests that these values must be realistic. It is surprising that the mean-square amplitude of the hydrogen atom normal to the molecular plane is considerably less than that of the oxygen atom. In view of this a 'riding' model (Busing & Levy, 1964) is invalid and it is not possible to get a meaningful correction of the O-H bond distance for thermal motion.

The assistance of Shri S. N. Momin and Shri H. Rajagopal in recording some of the data is gratefully

$$* R_w = [\omega(F_o^2 - F_c^2)^2 / \omega F_o^4]^{1/2}.$$

Table 2. Bond distances (in Å) and angles (in degrees)

The standard deviations are given in parentheses.

Bond	This work	Hodgson & Ibers	Δ/σ_p
C—C	1.5698 (17)	1.5740 (24)	1.4
C—O(1)	1.2559 (14)	1.2595 (16)	1.7
C—O(2)	1.2487 (15)	1.2473 (17)	0.6
O(1)—C—O(2)	125.89 (9)	126.27 (12)	2.5
C—C—O(1)	116.24 (10)	115.76 (14)	2.8
C—C—O(2)	117.86 (9)	117.96 (13)	0.6
O(3)—O(1)	2.7536 (17)	2.7602 (17)	2.7
O(3)—H	0.9628 (33)	0.80 (3)	5.4
H—H*	1.5541 (49)		
H—O(3)—H†	107.62 (33)	106.9 (32)	0.2
O(1)—O(3)—O(1)	118.82 (9)		
H—O(3)—O(1)	6.74 (14)		
O(3)—H—O(1)	169.66 (22)	165.9 (29)	1.3

* The angle from the *a* axis to the H—H vector measured away from the *c* axis is 46.45° (20).

† The dihedral angle between the H—O(3)—H and O(1)—O(3)—O(1) planes is 4.50° (17).

Table 3. Observed and calculated structure factors for $K_2C_2O_4 \cdot H_2O$ The nuclear scattering lengths used are (in units of 10^{-12} cm): potassium 0.373, carbon 0.675, oxygen 0.593 and hydrogen -0.378. The four columns in each set contain the Miller indices *h*, *l*, $100F_o^2$ and $100F_c^2$.

<i>k</i>	<i>h</i>	<i>l</i>	$100F_o^2$	$100F_c^2$
0	0	0	0	0
2	2388	2316	6 0	102 70
4	6951	6377	6 0	1340 1311
6	248	259	4 945	956 7 107 110
8	4717	4981	6 2273	2281 8 1644 1577
10	425	442	8 0	3418 3462 9 04 03
12	10107	10277	2 1265	1219 10 34 33
14	334	318	4 185	186 11 2572 2684
16	150	149	6 2273	2281 7 0 53 49
18	15889	15256	8 2	1277 1185 1 2057 2052
20	2560	2397	10 1	49 53 2 1277 1185
22	389	341	12 2	10107 10277 3 51 45
24	15020	14626	14 4	10715 9889 4 23 08
26	2407	2242	16 6	159 120 5 3285 3375
28	4318	4149	18 8	159 120 5 3285 3375
30	9523	9820	20 10	159 120 5 3285 3375
32	35	0	22 12	159 120 5 3285 3375
34	385	345	24 14	159 120 5 3285 3375
36	2930	2885	26 16	159 120 5 3285 3375
38	209	196	28 18	159 120 5 3285 3375
40	6571	6548	30 20	159 120 5 3285 3375
42	156	148	32 22	159 120 5 3285 3375
44	5842	5797	34 24	159 120 5 3285 3375
46	3210	3300	36 26	159 120 5 3285 3375
48	12803	13310	38 28	159 120 5 3285 3375
50	76	62	40 30	159 120 5 3285 3375
52	545	522	42 32	159 120 5 3285 3375
54	7843	7792	44 34	159 120 5 3285 3375
56	2035	1674	46 36	159 120 5 3285 3375
58	1626	1577	48 38	159 120 5 3285 3375
60	2249	2345	50 40	159 120 5 3285 3375
62	47	61	52 42	159 120 5 3285 3375
64	6254	6785	54 44	159 120 5 3285 3375
66	1626	1577	56 46	159 120 5 3285 3375
68	2169	2175	58 48	159 120 5 3285 3375
70	625	696	60 50	159 120 5 3285 3375
72	27	15	62 52	159 120 5 3285 3375

Table 4. Mean-square amplitudes of O(3) and H, in Å²

	U_{11}	U_{22}	U_{33}	U_{12}	U_{13}	U_{23}
O(3)	0.01988	0.02566	0.06434	0	0.00038	0
H	0.02853	0.03891	0.04428	0.00429	0.00290	0.00003

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